
Modeling Variability of Musculoskeletal Systems with Heteroscedastic Gaussian Processes

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Musculoskeletal systems have a number of benefits over traditional robots actuated by motors. To name only the most prominent: 1) better dexterity through higher accelerations[1], 2) embodied intelligence [3] and 3) account for higher accuracy demands by variable stiffness with altering co-contraction levels [2]. Although a number of replica of skeletal muscles exist - among them pneumatic artificial muscles (PAM) - they cannot be used in the same way in robots as in the human motor system. Rather, the beneficial properties of this particular actuator should be leveraged [1].

Here, we highlight an idea on how to quantify variability to subsequently alter it with a system actuated by antagonistic PAMs. The human motor system adjusts its compliance in unknown situation in case some external force perturbs the system. Additionally, it was observed that the change of compliance also influences the variability without considering any external force [2]. Humans decrease compliance by increasing the tension of all muscles in one joint, eventually ending up in higher co-contraction levels.

The idea of using co-contraction to change variability has been around for many years [6, 5]. Many of them try to account for variability with some kind of noise assumption in the model or learning a probabilistic model from data. Especially, Gaussian Processes (GP) have been used to model inverse and forward dynamics that describe the probability over next states \mathbf{s}' given the current state \mathbf{s} and action \mathbf{a} (forward) or $(\mathbf{s}, \mathbf{s}') \rightarrow \mathbf{a}$ (inverse). In both cases the posterior distribution of the GP has a mean function and a variance

$$\mu(\mathbf{x}_*) = k(X, \mathbf{x}_*)^T (K + \sigma_n^2 I)^{-1} \mathbf{y} \quad (1)$$

$$\sigma(\mathbf{x}_*) = k(\mathbf{x}_*, \mathbf{x}_*) + k(X, \mathbf{x}_*)^T (K + \sigma_n^2 I)^{-1} k(X, \mathbf{x}_*) \quad (2)$$

where $k(\cdot, \cdot)$ is a kernel function, X the design matrix with N training inputs as elements $\{\mathbf{x}_i^T\}_{i=1}^N$ and the corresponding targets $\mathbf{y} = \{y_i\}_{i=1}^N$. The test input \mathbf{x}_* as well as the training inputs \mathbf{x}_i consist in the context of dynamical systems of either a concatenation of the current state and action $[\mathbf{s}^T, \mathbf{a}^T]^T$ for forward dynamics or the current and desired consecutive state $[\mathbf{s}^T, \mathbf{s}'^T]^T$ for inverse dynamics.

We set up a toy example to illustrate our understanding of the connection between co-contraction, compliance, uncertainty and variability by fitting the function $f(x) = 1.5x^2 - 0.5x + 0.3 \sin(2\pi x) + 1$ with a GP from noisy samples. In particular, the target values $y = f(x) + \epsilon$ are corrupted by an input dependent noise term $\epsilon = \mathcal{N}(\mu, x^2\sigma)$. This should represent different variability levels in the state-action space as present in antagonistically actuated systems for different co-contraction levels.

Figure 1 depicts the fit with a GP that (a) assumes constant noise levels and (b) occupies a heteroscedastic noise treatment. The training data is generated at discrete values of x to point out what is meant by variability - similar but different outputs y at the same input locations x . The variance represents the uncertainty of how good the estimation of the model is at \mathbf{x}_* . Hence, for traditional GP regression the variance is lower where training data is available. This can be seen in figure 1(a) and more detailed in figure 2 within the interval $x = [0.3, 0.8]$. However, traditional GP regression does not discriminate the obvious different variability levels of intervals $x = [0, 0.3]$ and $x = [0.8, 1]$. Thus, variability cannot be modeled with the variance of a GP that uses Gaussian likelihood $p(\mathbf{y}|\mathbf{f}, X) = \mathcal{N}(\mathbf{f}, \sigma_n^2)$ where a constant σ_n is found by maximizing the marginal likeli-

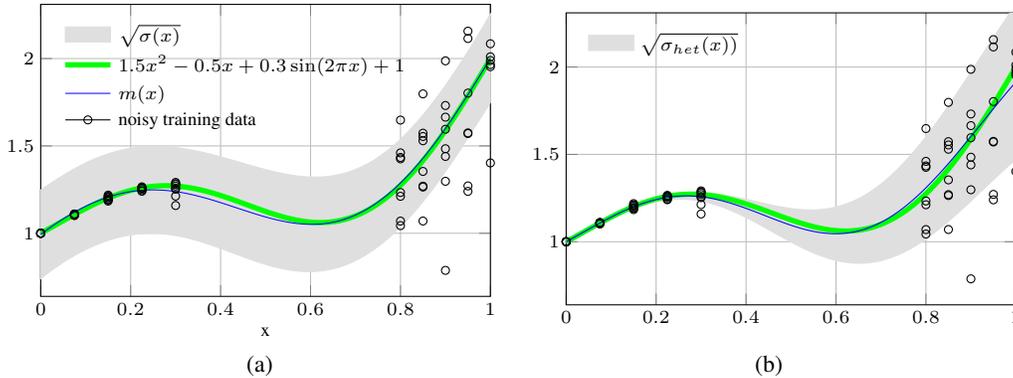


Figure 1: GP prediction with (a) constant & (b) heteroscedastic noise assumption.

hood for the complete dataset $D = \{X, \mathbf{y}\}$. The variability enters the variance of a GP only with the hyperparameters of the kernel function $k(\cdot, \cdot)$ as \mathbf{y} does not appear directly in eq. (2).

The solution we propose is to use heteroscedastic GPs instead. The approach used in figure 1(b) [4] adds a GP prior on σ_n hence learns the noise structure from data. Figure 2 confirms the ability of heteroscedastic GPs to account for different noise levels and hence qualifies to express variability in the data.

Having such a quantity is especially interesting for control of over-actuated systems. Among other applications, the variability can be incorporated into the cost function enabling to track a desired trajectory with a desired accuracy. This usage of variability could have advantageous implications: Robots need to be precise only in one point long a trajectory for many real-world tasks like table tennis or darts. Leaving aside the traditional trajectory tracking and all inherent difficulties to be precise at all times, allowing for higher variability before can possibly lead automatically to greater accuracy after at the point-of-interest. Additionally, high control signals are usually punished in cost functions to avoid bang-bang-control like actuation and therefore dangerous behaviors. Muscle-based systems reach equilibrium states also in case the controls are not run down to zero which is inherently different to motors. Hence, punishing control commands in musculoskeletal systems possibly renders good controls too costly. Variability could step in as a measure of how much activation is needed for a particular desired trajectory as it correlates with different co-contraction levels. If less exactness is required, less co-contraction is acceptable and thus lower controls will be applied.

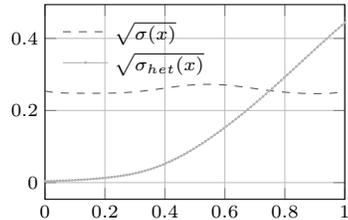


Figure 2: Standard deviations of the GPs in figure 1. The heteroscedastic GP of fig. 1(b) captures different noise levels of x while the traditional GP of (a) shows an increase in $\sigma(x)$ where no training data is available and does not distinguish between intervals $x = [0, 0.3]$ and $x = [0.8, 1]$.

Our subsequent work will be focused on supporting our claim of the presence and adjustability through co-contraction of variability in muscle-based systems, in simulation as well as on the real robot from [1].

References

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